

Theoretical background for observing ultra-slow microwaves in a Bose-Einstein condensate of alkali atoms

Yuriy V. Slyusarenko and Andrey G. Sotnikov

Akhiezer Institute for Theoretical Physics, NSC KIPT, 1 Akademicheskaya str., 61108 Kharkiv, Ukraine

We represent a new microscopic approach that allows studying the propagation properties of microwaves in a Bose-Einstein condensate of alkali atoms. It is assumed that the frequency of signal is tuned up to the transition between hyperfine ground state levels of such atoms. Pulse slowing conditions dependence on the system parameters is found. It is shown that the slowed signal can propagate in mentioned system with rather small energy loss. Such phenomenon is also studied in case of hyperfine levels Zeeman splitting. A possibility of ultra-slow microwaves observing in a condensed gas of cesium atoms is discussed.

Keywords: Green functions, alkali atoms, hyperfine structure, Bose-Einstein condensate, ultra-slow electromagnetic waves.

PACS: 05.30.-d; 03.75.Hh; 42.25.Bs.

1 Introduction

In 1999 the unique opportunity to observe the ultra-slow pulses of the optical region of spectra in a Bose-Einstein condensate (BEC) of sodium atoms was shown [1]. In this and subsequent experiments laser pulses were tuned up to dipole-allowed transitions.

Naturally, a question arises about the possibility of using another levels for observing electromagnetic pulses slowing in BEC of alkali atoms. In our opinion, it seems to be convenient to use microwave pulses tuned up to the transition between ground state hyperfine structure levels of hydrogen-like atoms. It is easy to see that such levels are rather stable. This fact, in particular, explains the universe abundance of microwaves with "famous" 21 cm wavelength, which correspond to the transitions between hyperfine states of the atomic hydrogen. Moreover, there are many trapping-related experiments with ultra-cold atoms, which are prepared in different hyperfine states (see, *e.g.*, experiments for the multi-component BEC observing [2]).

However, as it is easy to see, alkali atoms in the ground state do not have a dipole moment. As a result, theoretical descriptions (see, *e.g.*, ref. [3]), in which atoms are considered as dipoles, become no longer convenient. For our opinion, the most appropriate way to go out from this situation is using a microscopic approach. Such an approach, for example, was developed in ref. [4]. It is based on the approximate formulation of the second quantization method and takes into account the presence of bound states of particles. As it is shown below, such method is universal in some respect and allows to describe the response of the system not only to microwave range perturbation (the signal tuned up to the transitions between hyperfine ground state

levels), but also to the optical range perturbation (the pulse tuned up to the dipole-allowed transitions) [5].

Let us briefly state out some basic principles of the suggested approach.

2 Theoretical basis

As one can suggest, alkali atoms are similar to the hydrogen atom by the internal structure, and can be considered as bound states of two "elementary" particles (valence electron and atomic core). From the standpoint of the quantum mechanics, it is also known that a bound state energy can lie not only in the discrete spectrum region, but also in the continuum one. This fact was taken into account in ref. [4] in construction of novel formulation of the second quantization method in the presence of bound states (atoms). It was also used in ref. [5] in construction of microscopic theory of the hydrogen-like low temperature plasma response to the external electromagnetic field.

Thus, to construct a microscopic approach one needs to consider a system, that consists of two kinds of free fermions (ions and electrons) and their bound states (alkali atoms), as it was done in ref. [4]. But, to simplify the further description, we have to note the following. In recent theoretical investigations [6] of hydrogen-like low temperature plasma in an equilibrium state it was shown that the density of free fermions is exponentially small in comparison with the density of atoms.

Therefore, to build microscopic approach describing electrodynamic processes at low temperatures one can use the mentioned second quantization method with preserving the quantities that correspond only to bound states (atoms) contribution. According to ref. [4], the system Hamiltonian can be written in the

following form:

$$\hat{\mathcal{H}}(t) = \hat{\mathcal{H}}_0 + \hat{\mathcal{H}}_{\text{int}} + \hat{V}(t), \quad \hat{\mathcal{H}}_0 = \hat{\mathcal{H}}_{\text{ph}} + \hat{\mathcal{H}}_{\text{p}}, \quad (1)$$

where $\hat{\mathcal{H}}_{\text{ph}}$ is the Hamiltonian for free photons and $\hat{\mathcal{H}}_{\text{int}}$ is the Hamiltonian of interaction between atoms (we neglect Hamiltonians $\hat{\mathcal{H}}_{\text{ph}}$ and $\hat{\mathcal{H}}_{\text{int}}$ below, but for the explicit form see also ref. [4]).

The operator $\hat{\mathcal{H}}_{\text{p}}$ in eq. (1) is the Hamiltonian for free particles (atoms)

$$\hat{\mathcal{H}}_{\text{p}} = \sum_{\alpha} \int d\mathbf{X} \left\{ \frac{1}{2m} \frac{\partial \hat{\eta}_{\alpha}^{\dagger}(\mathbf{X})}{\partial \mathbf{X}} \frac{\partial \hat{\eta}_{\alpha}(\mathbf{X})}{\partial \mathbf{X}} + \varepsilon_{\alpha} \hat{\eta}_{\alpha}^{\dagger}(\mathbf{X}) \hat{\eta}_{\alpha}(\mathbf{X}) \right\}, \quad (2)$$

where $\hat{\eta}_{\alpha}^{\dagger}(\mathbf{X})$, $\hat{\eta}_{\alpha}(\mathbf{X})$ are the creation and annihilation operators of hydrogen-like (alkali) atoms with the set of quantum numbers α at the point \mathbf{X} ; ε_{α} is the energy of an atom in the state characterized by quantum numbers α ; m is the atomic mass.

And, finally, the operator $\hat{V}(t)$ in eq. (1) represents the Hamiltonian that describes the interaction of particles with the electromagnetic field

$$\hat{V}(t) = -\frac{1}{c} \int d\mathbf{x} \mathbf{A}^{(e)}(\mathbf{x}, t) \hat{\mathbf{j}}(\mathbf{x}) + \int d\mathbf{x} \varphi^{(e)}(\mathbf{x}, t) \hat{\sigma}(\mathbf{x}),$$

where $\mathbf{A}^{(e)}(\mathbf{x}, t)$ and $\varphi^{(e)}(\mathbf{x}, t)$ are the vector and scalar potentials of the external electromagnetic field, respectively, and operators $\hat{\mathbf{j}}(\mathbf{x})$ and $\hat{\sigma}(\mathbf{x})$ are the charge and current density operators, respectively:

$$\begin{aligned} \hat{\sigma}(\mathbf{x}) &= \frac{1}{\mathcal{V}} \sum_{\mathbf{p}, \mathbf{p}'} \sum_{\alpha, \beta} e^{i\mathbf{x}(\mathbf{p}' - \mathbf{p})} \\ &\quad \times \sigma_{\alpha\beta}(\mathbf{p} - \mathbf{p}') \hat{\eta}_{\alpha}^{\dagger}(\mathbf{p}) \hat{\eta}_{\beta}(\mathbf{p}'), \\ \hat{\mathbf{j}}(\mathbf{x}) &= \frac{1}{\mathcal{V}} \sum_{\mathbf{p}, \mathbf{p}'} \sum_{\alpha, \beta} e^{i\mathbf{x}(\mathbf{p}' - \mathbf{p})} \left(\mathbf{I}_{\alpha\beta}(\mathbf{p} - \mathbf{p}') \right. \\ &\quad \left. + \frac{(\mathbf{p} + \mathbf{p}')}{2M} \sigma_{\alpha\beta}(\mathbf{p} - \mathbf{p}') \right) \hat{\eta}_{\alpha}^{\dagger}(\mathbf{p}) \hat{\eta}_{\beta}(\mathbf{p}'). \end{aligned} \quad (3)$$

Here \mathcal{V} is the system volume. Note that the charge and current density matrix elements in eq. (3) can be expressed (see ref. [4]) in terms of atom wave functions $\varphi_{\alpha}(\mathbf{x})$:

$$\begin{aligned} \sigma_{\alpha\beta}(\mathbf{k}) &= e \int d\mathbf{y} \varphi_{\alpha}^*(\mathbf{y}) \varphi_{\beta}(\mathbf{y}) \\ &\quad \times \left[\exp\left(i \frac{m_{\text{p}}}{m} \mathbf{k} \mathbf{y}\right) - \exp\left(-i \frac{m_{\text{e}}}{m} \mathbf{k} \mathbf{y}\right) \right], \\ \mathbf{I}_{\alpha\beta}(\mathbf{k}) &= -\frac{ie}{2} \int d\mathbf{y} \\ &\quad \times \left(\varphi_{\alpha}^*(\mathbf{y}) \frac{\partial \varphi_{\beta}(\mathbf{y})}{\partial \mathbf{y}} - \frac{\partial \varphi_{\alpha}^*(\mathbf{y})}{\partial \mathbf{y}} \varphi_{\beta}(\mathbf{y}) \right) \\ &\quad \times \left[\frac{1}{m_{\text{p}}} \exp\left(i \frac{m_{\text{e}}}{m} \mathbf{k} \mathbf{y}\right) + \frac{1}{m_{\text{e}}} \exp\left(-i \frac{m_{\text{p}}}{m} \mathbf{k} \mathbf{y}\right) \right], \end{aligned} \quad (4)$$

where e is the electron charge absolute value, m_{p} and m_{e} are the atomic core and electron mass, respectively ($m = m_{\text{p}} + m_{\text{e}}$).

3 System response to the external electromagnetic field in the framework of Green functions formalism

Non-relativistic equations of quantum electrodynamics, which were found in ref. [4] on the basis of Hamiltonians (1)-(4) and which, in turn, were used in ref. [5], allow to study the linear response of the system to a perturbation by the external electromagnetic field [7]. To find out such a response it is most convenient to use the Green functions formalism (as it was done in ref. [5]). In the framework of this formalism the scalar (charge) Green function can be defined as:

$$G^{(+)}(\mathbf{x}, t) = -i\theta(t) \text{Sp} w[\hat{\sigma}(\mathbf{x}, t), \hat{\sigma}(0)],$$

where $\theta(t)$ is the Heaviside function, w is the Hibbs distribution operator and charge density operators (see definition (3)) must be taken in the Heisenberg representation. Neglecting the interaction between particles (as it can be done for dilute gases), one can get the expression for the scalar Green function Fourier transform (see ref. [5]):

$$\begin{aligned} G^{(+)}(\mathbf{k}, \omega) &= \frac{1}{\mathcal{V}} \sum_{\mathbf{p}} \sum_{\alpha, \beta} \sigma_{\alpha\beta}(\mathbf{k}) \sigma_{\beta\alpha}(-\mathbf{k}) \\ &\quad \times \frac{f_{\alpha}(\mathbf{p} - \mathbf{k}) - f_{\beta}(\mathbf{p})}{\varepsilon_{\alpha}(\mathbf{p}) - \varepsilon_{\beta}(\mathbf{p} - \mathbf{k}) + \omega + i0}, \end{aligned} \quad (5)$$

where $\varepsilon_{\alpha}(\mathbf{p}) = \varepsilon_{\alpha} + \mathbf{p}^2/2m$, $f_{\alpha}(\mathbf{p})$ is the bosonic distribution function of the ideal gas of hydrogen-like (alkali) atoms

$$f_{\alpha}(\mathbf{p}) = \{\exp[(\varepsilon_{\alpha}(\mathbf{p}) - \mu_{\alpha})/T] - 1\}^{-1}.$$

Here μ_{α} is the chemical potential of atoms in the state α , T is the temperature of the gas that is taken in energy units.

Using the developed theory, it is not difficult to find the permittivity of such gas at low temperatures. From eq. (5) one gets (see also ref. [5]):

$$\begin{aligned} \varepsilon^{-1}(\mathbf{k}, \omega) &= 1 + \frac{4\pi}{k^2} \frac{1}{\mathcal{V}} \sum_{\mathbf{p}} \sum_{\alpha, \beta} \sigma_{\alpha\beta}(\mathbf{k}) \sigma_{\beta\alpha}(-\mathbf{k}) \\ &\quad \times \frac{f_{\alpha}(\mathbf{p} - \mathbf{k}) - f_{\beta}(\mathbf{p})}{\varepsilon_{\alpha}(\mathbf{p}) - \varepsilon_{\beta}(\mathbf{p} - \mathbf{k}) + \omega + i0}. \end{aligned} \quad (6)$$

As it is known, at extremely low temperatures a Bose-Einstein condensate (BEC) of alkali atoms can be formed. At temperatures much lower the critical point temperature $T \ll T_0$ the atomic distribution functions $f_{\alpha}(\mathbf{p})$ are proportional to the Dirac delta-function $\delta(\mathbf{p})$. Note also that at temperatures $T \lesssim T_0$ the chemical potential μ_{α} does not depend on temperature and must be set equal to energy of the lowest level of atoms, which form BEC (see refs. [6, 8]). Therefore,

after integration of eq. (6) over momentum \mathbf{p} the expression for the permittivity of the studied gas in BEC state ($T \rightarrow 0$) takes the form:

$$\epsilon^{-1}(\mathbf{k}, \omega) \approx 1 + \frac{1}{2\pi^2 k^2} \sum_{\alpha, \beta} \sigma_{\alpha\beta}(\mathbf{k}) \sigma_{\beta\alpha}(-\mathbf{k}) \times \left[\frac{\nu_\alpha}{\omega + \Delta\varepsilon_{\alpha\beta} - \varepsilon_k + i\gamma_{\alpha\beta}} - \frac{\nu_\beta}{\omega + \Delta\varepsilon_{\alpha\beta} + \varepsilon_k + i\gamma_{\alpha\beta}} \right], \quad (7)$$

where ν_α is the density of condensed atoms in the quantum state α , $\varepsilon_k = k^2/2M$, and the quantities $\sigma_{\alpha\beta}(\mathbf{k})$ are still defined by the formula (4). Note that due to damping processes in real systems we also introduce the linewidth $\gamma_{\alpha\beta}$, concerned with the transition probability between the state α and state β . As it easy to see, in eq. (7) at frequencies close to the energy interval $\Delta\varepsilon_{\alpha\beta}$ ($\Delta\varepsilon_{\alpha\beta} \equiv \varepsilon_\alpha - \varepsilon_\beta$) some peculiarities appear. In fact, such a behavior must have a strong impact on the dispersion characteristics of the gas and can be an underlying condition for pulses slowing.

To study the propagation properties of the signal one should add the dispersion relation for free waves, that can spread in the studied system:

$$\frac{\omega^2}{c^2} \epsilon(\mathbf{k}, \omega) \mu(\mathbf{k}, \omega) - k^2 = 0, \quad (8)$$

where $\mu(\mathbf{k}, \omega)$ is the magnetic permeability that can be found analogically in the framework of Green functions formalism (see in that case ref. [5]).

4 Microwaves slowing caused by hyperfine structure levels

In the previous section the expression for the permittivity of the system in BEC state (see eq. (7)) was represented. For the system with the frequency of the external field tuned up to the difference between two defined levels (marked below by subscripts 1 and 2) it can be written in a more suitable form:

$$\epsilon^{-1}(\mathbf{k}, \omega) \approx 1 + \frac{g_1 g_2 |\sigma_{12}(\mathbf{k})|^2 (\nu_1 - \nu_2)}{2\pi^2 k^2 (\delta\omega + i\gamma)}. \quad (9)$$

Here g_j is the degeneracy order over total spin of j level ($j = 1, 2$), $\delta\omega = \omega - \Delta\varepsilon_{21}$ is the laser detuning ($|\delta\omega| \ll \Delta\varepsilon_{21}$), $\gamma \equiv \gamma_{12}$ is linewidth related to the transition probability from the upper to lower state. Note that we also neglected the term ε_k (see eq. (7)), the substantiation of such operation is discussed below.

To study the propagation properties it is more convenient to turn to the refractive index and the damping factor quantities. To do it we set the magnetic permeability in the dispersion relation (8) close to unity, $\mu(\mathbf{k}, \omega) = 1$. In this case the refractive index $n(\mathbf{k}, \omega)$ and the damping factor $\chi(\mathbf{k}, \omega)$ can be expressed in

terms of the real and imaginary part of the permittivity (ϵ' and ϵ'' , respectively), which in turn can be derived from eq. (9):

$$\epsilon' = \frac{\delta\omega(\delta\omega + a) + \gamma^2}{(\delta\omega + a)^2 + \gamma^2}, \quad \epsilon'' = \frac{\gamma a}{(\delta\omega + a)^2 + \gamma^2}, \quad (10)$$

where

$$a(\mathbf{k}) = (\nu_1 - \nu_2) \frac{g_1 g_2 |\sigma_{12}(\mathbf{k})|^2}{2\pi^2 k^2}. \quad (11)$$

Now one can find the dependence of group velocity on system parameters. As it is known, the group velocity of a propagating pulse can be defined as:

$$v_g = \frac{c}{n + \omega(\partial n / \partial \omega)}.$$

Thus, after some mathematical transformations, in case of energy dissipation smallness and strong dispersion, one gets:

$$v_g \approx 2c \frac{((\delta\omega + a)^2 + \gamma^2)^2}{a\omega [(\delta\omega + a)^2 - \gamma^2]}. \quad (12)$$

This expression gives the opportunity to study the dependence of the group velocity not only on detuning $\delta\omega$ and linewidth γ , but also on characteristic properties of the system under consideration that have an impact on the parameter a value (see eq. (11)).

Now let us obtain conditions, in which the ultra-slow microwaves phenomenon for two-level system in BEC state can be observed. To this end we proceed to the limit $\delta\omega \rightarrow 0$. In this case, according to eq. (10), the real and imaginary parts of permittivity tend to the limits:

$$\lim_{\delta\omega \rightarrow 0} \epsilon' = \frac{\gamma^2}{\gamma^2 + a^2}, \quad \lim_{\delta\omega \rightarrow 0} \epsilon'' = \frac{\gamma a}{\gamma^2 + a^2}.$$

Hence, the dissipation smallness condition ($\epsilon' \ll |\epsilon''|$) can be written as follows:

$$\frac{|a|}{\gamma} \ll 1.$$

Note that it is necessary also to add the slowing down (strong dispersion) condition, which in this case (see eq. (12)) can be written as:

$$\frac{c}{v_g} \approx \frac{\Delta\varepsilon_{21} |a|}{2\gamma^2} \gg 1,$$

For the defined system with a fixed energy structure ($\Delta\varepsilon, \gamma = \text{const}$) the only parameter that can be varied is the occupation difference $(\nu_1 - \nu_2)$, which is included in the parameter a (see eq. (11)). Thus, basing on these relations we get the expression that characterizes the region where the mentioned phenomenon can be observed:

$$\frac{\gamma}{\Delta\varepsilon_{21}} \ll \frac{|a|}{\gamma} \ll 1, \quad (13)$$

By considering the example of the hyperfine levels of the ground state for cesium atoms let us demonstrate that such region can exist. The choice of such levels, as it is mentioned above, is stimulated by their stability and pumping capability. Note also that for such levels the dipole transitions are forbidden, thus transitions come from the higher order effects that result in extremely small values of linewidths. It is shown below that such fact gives the opportunity for a signal to propagate with a small loss of energy.

It should be mentioned that the description can be extended to other hydrogen-like atoms and other type of levels analogically. For example, it can be used for a description of experiments with an ultra-slow light in BEC of sodium atoms [1], in which the dipole-excited states were used for a pulse slowing. In this case to get numerical results from the developed approach one can restrict calculations of the charge density matrix element σ_{12} (see eq. 4) to the first order approximation over $(\mathbf{k}\mathbf{y}) \ll 1$:

$$\sigma_{12}^{(1)}(\mathbf{k}) \approx i\mathbf{k}\mathbf{d}_{12},$$

with the dipole moment \mathbf{d}_{12} that corresponds to the dipole transition $1 \rightarrow 2$:

$$\mathbf{d}_{12} = e \int \mathbf{y} d\mathbf{y} \varphi_1^*(\mathbf{y}) \varphi_2(\mathbf{y}).$$

But, it is well known that alkali metals (^{133}Cs in particular) in the ground state do not have a dipole moment \mathbf{d} , thus to describe the pulse slowing on dipole-forbidden transitions the charge density matrix element σ_{12} (see definition (4)) must be expanded to the second order over $(\mathbf{k}\mathbf{y}) \ll 1$. As a result, one gets:

$$\sigma_{12}^{(2)}(k) \approx \frac{e}{3}(kr_0)^2, \quad (14)$$

where r_0 is the atomic radius (for cesium ground state $r_0 \approx 2,6 \times 10^{-8}$ cm [9]), e is the electron charge. Taking $g_1 = 7$, $g_2 = 9$ ($g_j = 2F_j + 1$, F_j is the total spin of an atom in j hyperfine state, $j = 1, 2$), $k = (\Delta\varepsilon_{21}/c)$, where $\Delta\varepsilon_{21} \approx 3,8 \times 10^{-5}$ eV (microwaves with frequency 9,1926 GHz), the linewidth $\gamma \approx 3,8 \times 10^{-21}$ eV, corresponding to the anticipated accuracy (10^{-16}) in "cesium fountain clock" experiments [10], and basing on the expressions (11), (13) one can find the region for the occupation difference, in which the pulse slowing phenomenon can be observed:

$$10^{-3} \text{ cm}^{-3} \ll |\nu_1 - \nu_2| \ll 3 \times 10^{13} \text{ cm}^{-3}. \quad (15)$$

From this inequality one can conclude that the effect becomes greater with the density difference increasing until it reaches the upper limit of the expression (15), when damping effects prevail in the system. We should stress that the region of densities (15) looks convenient from the standpoint of experiments for BEC regime (see, *e.g.*, the experiment [11], in which cesium atoms with the density $\nu = 7 \times 10^{10} \text{ cm}^{-3}$ were used).

We should also note the following. As it easy to see from eq. (12) in the limit $\delta\omega \rightarrow 0$, the sign of the group velocity v_g depends directly on the sign of the quantity a , that in turn depends on the sign of the difference $(\nu_1 - \nu_2)$. In other words, it depends on whether the population is normal or inverse.

In case of normal population ($\nu_1 > \nu_2$) the group velocity of signal is negative. It is traditionally considered that the group velocity for the transparent matter is positive. But here one can conclude that due to the relation (13), the signal can propagate in the system with rather small dissipation (*i.e.*, in fact, the matter is transparent) and rather slow velocity. Let us note that an observing of electromagnetic pulses with negative group velocity is not so abnormal. The existence of such kind of phenomena for physical systems when the wave frequency is close to atomic (or molecular) resonances was pointed out and studied in many works (both theoretical, see *e.g.* [12], and experimental [13, 14]). In case of inverse population ($\nu_1 < \nu_2$) more "normal" situation takes place because the group velocity of the slowed pulse is positive.

We stress that such rather unusual phenomenon occurs due to the unique property of hyperfine splitted ground state levels. One can show that for a two-level system with allowed dipole transitions the pulse will not propagate due to a large absorption. The most obvious way to go out from this situation is the second (coupling) laser usage that leads to the electromagnetically induced transparency (EIT, a detailed description see in ref. [15]).

Now, let us say a few words about the quantity ε_k , which was neglected in deriving the equation (9). If we assume $k = (\Delta\varepsilon_{21}/c)$, one can find that, *e.g.*, for cesium, $\varepsilon_k \approx 3,5 \times 10^{-30}$ eV. So, even at the point $\delta\omega = 0$ it is small in comparison with the linewidth $\gamma = 3,8 \times 10^{-21}$ eV. Thus, we proved that the used approximation is correct.

5 Pulse slowing concerned with Zeeman splitted hyperfine structure levels

In the previous section the observing possibility for ultra-slow microwaves caused by hyperfine structure levels was shown. There we assumed that such levels are degenerate, *i.e.* the external magnetic field is absent. But the presence of an external magnetic field (such situation occurs in most of experiments) results in more complicated picture. Each of hyperfine structure levels splits to additional components with different total spin projection m_F (*e.g.* for cesium atoms see fig. 1). As it is well known, in a weak external magnetic field the energy difference between such components is proportional to the magnetic field intensity (the Zeeman effect).

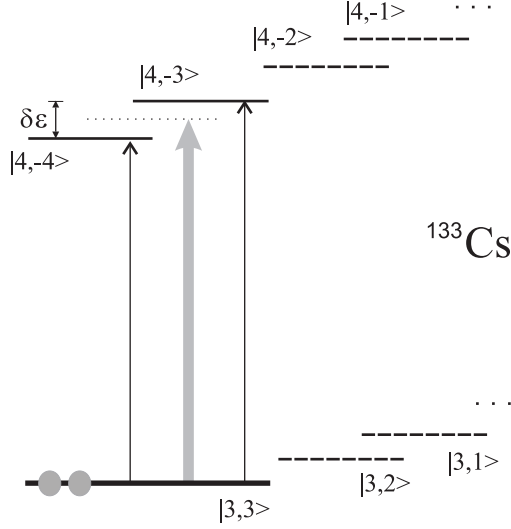


Figure 1: Ground state energy structure of cesium atom in the external magnetic field. The first sign in brackets corresponds to the total atomic spin F , the second one to its projection m_F . For good visibility and compactness not all sublevels are showed.

If the components of the upper hyperfine structure level are kept away from each other (the linewidth of levels is much less than the energy difference between levels of multiplet $\delta\varepsilon$, as shown in fig. 1), the signal tuned up exactly to the transition between the occupied lower state and one of the upper states (narrow black arrowed lines in fig. 1) can be slowed down, as it is pointed out in the previous section. The only difference is that the states are no more degenerated ($g_1 = g_2 = 1$) and the linewidth γ_j in eq. (7) corresponds to the transition probability from the chosen component with the projection $j = (-F', \dots, F')$ to the occupied lower state. Taking all other values the same as when deriving the inequality (15) in the presence of the external magnetic field one can find:

$$10^{-1} \text{cm}^{-3} \ll |\nu_1 - \nu_2| \ll 2 \times 10^{15} \text{cm}^{-3}. \quad (16)$$

In the experiment [16] for the BEC regime cesium atoms with a peak density of $1,3 \times 10^{13} \text{cm}^{-3}$ were kept in a trap. Thus, according to the inequality (16), one can conclude that in such system the ultra-slow microwaves phenomenon could be observed. Moreover, direct calculations show that the group velocity of the signal propagating in the mentioned system is close to $9 \times 10^{-6} \text{m/s}$. It means that the pulse tuned up in such way covers the distance of 1 millimeter in the condensed cesium vapour nearly in 2 minutes!

Let us note that to observe such a kind of phenomenon one needs to use the microwave signal with the linewidth γ_s much less than the level linewidth

$$\gamma_s \ll \gamma_j. \quad (17)$$

Therefore, one can meet essential experimental difficulties because, as it was mentioned earlier, the linewidth

concerned with the transition between hyperfine structure levels is sufficiently small.

However, in the presence of an external magnetic field there are more regions where slowing down phenomenon can be observed. For example, the microwave signal, detuned relatively to the transitions between two neighbour states of the upper hyperfine multiplet and occupied lower state (wide grey arrow line on fig. 1) can also be slowed down to sufficiently small values. In that case the group velocity to a greater extent depends on the magnetic field intensity and the condition (17) is not necessary.

On the basis of the developed approach it is not difficult also to describe such variant of slowing, taking into account the Zeeman splitting. If we assume that all atoms are condensed in lower (occupied) state with the density ν and energy ε_0 , we get (see eq. (7)):

$$\epsilon^{-1}(\mathbf{k}, \omega) \approx 1 + \frac{\nu |\sigma_{12}(\mathbf{k})|^2}{2\pi^2 k^2} \times \sum_{j=-F'}^{F'} \left[\frac{1}{\omega - \Delta\varepsilon_j(H) + i\gamma_j} \right], \quad (18)$$

where $\sigma_{12}(\mathbf{k})$ can be found from the expression (14) and $\Delta\varepsilon_j = (\varepsilon_{|F', m_{F'}>} - \varepsilon_0)$. Now, using the derived equation, one can study the dependence of the refractive index on the frequency of microwave signal and its slowing down conditions.

Let us demonstrate it also by the cesium atoms example. The dependence of the refractive index in case of the external magnetic field presence has a rather complicated form due to a large number of splitted levels (9 levels in the upper state with $F = 4$), thus it is more convenient to show such dependence in a figure. In fig. 2 one can see that in central part ($\delta\omega = 0$ that corresponds to the wide grey arrowed line in fig. 1) the refractive index has a slope with steepness depending on the distance $\delta\varepsilon$ between the levels $|4, -4\rangle$ and $|4, -3\rangle$. It means that the microwave signal, which is detuned in such way, can also be slowed. The direct calculations show that, *e.g.* for the ideal gas of cesium atoms in BEC state ($\nu = 1,3 \times 10^{13} \text{cm}^{-3}$, $\gamma_j = 3,8 \times 10^{-21} \text{eV}$) with the energy of splitting $\delta\varepsilon = 3,8 \times 10^{-18} \text{eV}$ ($\delta\varepsilon/\gamma_j = 10^3$, such as showed on fig. 2), the pulse can be slowed down to 1,07 m/s.

6 Conclusion

Thus, by means of the microscopic approach we studied the linear response of an ideal atomic gas to disturbing effect of an external electromagnetic field. Our approach was based on a novel formulation of the second quantization method in the presence of bound states of particles [4]. The use of such an approach allowed us to obtain expressions for the dielectric permittivity of dilute gases of alkali atoms in BEC state. The existence

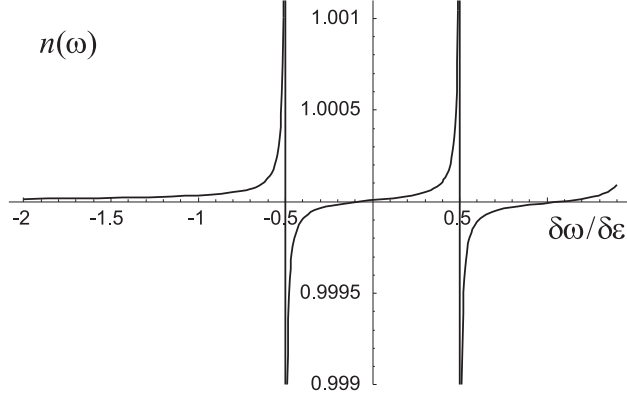


Figure 2: Refractive index dependence for cesium atoms in BEC state in the region of frequencies close to the transitions between hyperfine splitted levels. Left and right narrow peaks correspond to the transitions $|3, 3\rangle \rightarrow |4, -4\rangle$ and $|3, 3\rangle \rightarrow |4, -3\rangle$, respectively. Note that with frequency increasing (to the right side from the figure edge) the behavior qualitatively repeats. The used parameter for the Zeeman splitting energy is $\delta\epsilon/\gamma_j \approx 10^3$.

of resonance frequencies in expression for permittivity was found.

Our approach raised a possibility to study propagation properties of the microwave signal, tuned up to the transition between two hyperfine ground state levels of alkali atoms that were considered in BEC state. In contrast to dipole-allowed transitions, it was demonstrated that the pulse could propagate in such system with rather small energy loss. Due to this fact we introduced the group velocity concept. The slowing down conditions for the signal that propagated in BEC (at the limit of zero temperatures) were studied. Moreover, we revealed the dependence of the group velocity sign on the level's population difference. Thus, we suggested that in some cases in a Bose-Einstein condensate could propagate the weak-damping microwaves with a negative group velocity.

In case of the external static magnetic field (we took into account the Zeeman splitting) the ultra-slow microwaves phenomenon was also studied. Considering the example of cesium atoms vapour it was shown that in some conditions the pulse could be slowed down to extremely small values.

References

- [1] Hau L., Harris S., Datton Z., and Behvoozi C. *Nature* **297** (1999), 594.
- [2] Matthews M., Hall D., Jin D., Ensher J., Wieman C., Cornell E., Dalfovo F., Minniti C., and Stringari S. *Phys. Rev. Lett.* **81** (1998), 243.
- [3] Allen L. and Eberly J. *Optical Resonance and Two-Level Atoms*. Dover, New York (1987).
- [4] Peletminskii S. and Slyusarenko Yu. *J. Math. Phys.* **46** (2005), 022301; quant-ph/0605159.
- [5] Slyusarenko Yu. and Sotnikov A. *Cond. Matt. Phys.* **9** (2006), 459; cond-mat/0702637.
- [6] Slyusarenko Yu. and Sotnikov A. *Low Temp. Phys.* **33** (2007), 30.
- [7] Akhiezer A. and Peletminskii S. *Methods of Statistical Physics*. Pergamon, Oxford (1981).
- [8] Akhiezer A., Peletminskii S., and Slyusarenko Yu. *JETP* **86** (1998), 501.
- [9] Clementi E., Raimondi D., and Reinhardt W. J. *Chem. Phys.* **38** (1963), 2686.
- [10] Clairon A., Salomon C., Guellati S., and Phillips W. *Europhys. Lett.* **12** (1991), 683.
- [11] Guéry-Odelin D., Söding J., Desbiolles P., and Dalibard J. *Europhys. Lett.* **44** (1998), 25.
- [12] Kadomtsev B. *Collective phenomena in plasmas*. Pergamon, New York (1978).
- [13] Chu S. and Wong S. *Phys. Rev. Lett.* **48** (1982), 738.
- [14] Segard B. and Macke B. *Phys. Lett.* **109A** (1985), 213.
- [15] Harris S. *Physics Today* **50** (1997), 36.
- [16] Weber T., Herbig J., Mark M., Nägerl H., and Grimm R. *Science* **299** (2003), 232.